Knowledge for teaching primary mathematics: More than meets the eye

Tim Rowland

Advisory External Evaluator, *Enhancing Mathematical Learning through Talk*

3rd July 2015

University of East Anglia

HiST Trondheim

University of Cambridge

Department for Education

Brunel UNIVERSITY LONDON

Enhancing Mathematical Learning through Talk

3rd July 2015

SUPPORTED BY

MAYOR OF LONDON

The Brighter Borough Wandsworth

merton

Merton Education Partnership
The Education Network

I want to talk about *what primary mathematics teachers (need to) know.*

Professional knowledge base - things that (these) professionals know, that other educated persons generally don't know - and don't need to know.

e.g. lawyers, medical practitioners, computer scientists, priests, … , plumbers, farmers …

What about teachers (of mathematics, at all educational levels)?

Public perception in the UK about mathematics teachers' knowledge –

- Secondary mathematics teachers know mathematics (and have a kind of empathy with learners which enables them to 'put it across')

- Primary mathematics teachers don't need to know much (mathematics) because it's all very easy …

- Both beliefs are mistaken!

A provocation -

The true story of Jason and Elliot’s quarters

Jason, the teacher, was reviewing halves and quarters with a Year 3 class, to prepare for introducing other fractions. The pupils each had a small oblong whiteboard, and a dry-wipe pen. First, Jason asked them to “split” their individual whiteboards into two, and discussed their responses with the class.

(short video here)

Question:

What does Jason need to know in order to make an adequate and helpful response to Elliot’s contribution?

But first …

In the beginning ... Lee Shulman (1987)

Seven categories of teacher knowledge:

GENERIC KNOWLEDGE

- general pedagogical knowledge - generic principles of classroom management;
- knowledge of learners;
- knowledge of educational contexts, communities and cultures;
- knowledge of educational purposes and values.

CONTENT KNOWLEDGE ('THE MISSING PARADIGM')

- *subject matter knowledge* (SMK) – product *and* process;
- *pedagogical content knowledge* (PCK) - includes forms of representation of concepts, useful analogies, examples, demonstrations;
- *curriculum knowledge* - materials and programmes.

Lee Shulman, L. S. (1987) 'Knowledge and teaching: Foundations of the new reform.' *Harvard Educational Review*, 57(1), pp. 1-22

Subject matter knowledge (SMK)

Knowledge of the content and organisation of the discipline *per se*.

SMK consists of *two distinct and distinctive* kinds of knowledge [c.f. content and process]

1. substantive knowledge - the key facts, concepts, principles and structures/explanatory frameworks in a discipline

2. syntactic knowledge - the rules of evidence and warrants of truth within that discipline, *the nature of enquiry* in the field, and how new knowledge is introduced and accepted in that community – *how to find out*

(Schwab, 1978; Shulman and Grossman, 1988)

Syntactic knowledge: an example

A class of 9- and 10-year-olds were asked to give a fraction between ½ and ¾.

The teacher expected the average, $\frac{5}{8}$

One girl answered ⅔. The teacher asked how she knew that ⅔ lies between ½ and ¾. The girl explained:

"Because 2 is between the 1 and the 3, and on the bottom the 3 lies between the 2 and the 4".

Imagine you are the teacher. How would you respond?

Rowland, T. and Zazkis, R. (2013) Contingency in the mathematics classroom: Opportunities taken and opportunities missed. *Canadian Journal of Science, Mathematics and Technology Education* 13(2), pp. 137–153.

Pedagogical content knowledge (PCK)

The link between knowing something *for oneself* and being able to enable *others* to come to know it.

Includes

1. ways of *representing* the subject which makes it comprehensible to others …

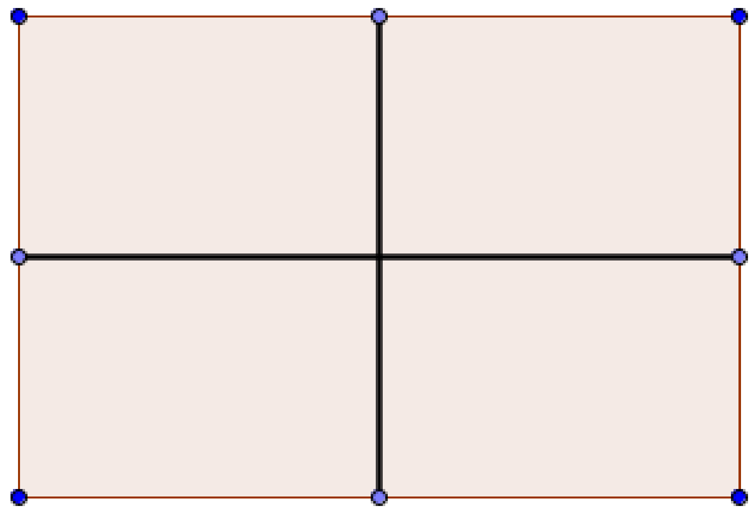
2. use of analogies, illustrations, *examples*, explanations and demonstrations –

3. understanding what makes the learning of specific topics easy or difficult [common misconceptions etc]

Importance of *unpacking* (Ball) – recognising the complexity of ‘compressed’, taken-for-granted knowledge.

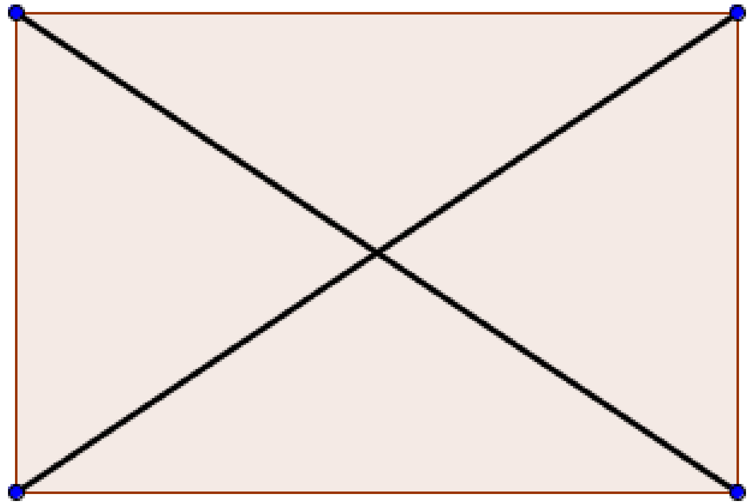
Now let's go back to Jason's classroom …

Jason had asked the class to split their boards "into four". Most children drew two lines parallel to the sides



Rebecca's quarters

… but Elliot drew the two diagonals …



Jason: What has Elliot done that is different to what Rebecca has done?

Sophie: Because he's done the lines diagonally.

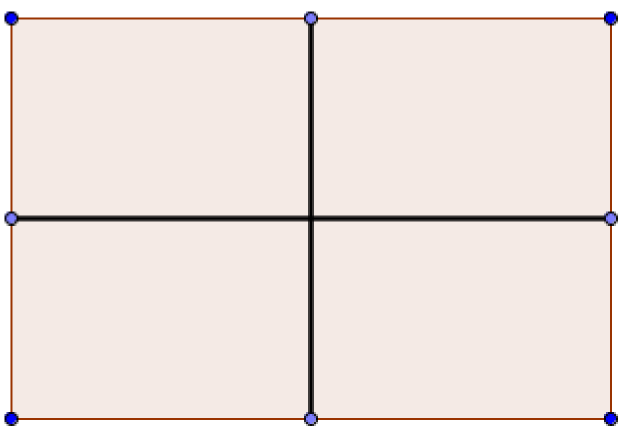
The pupils agree that Rebecca's board "has been split equally". Jason elicits the word 'quarters'.

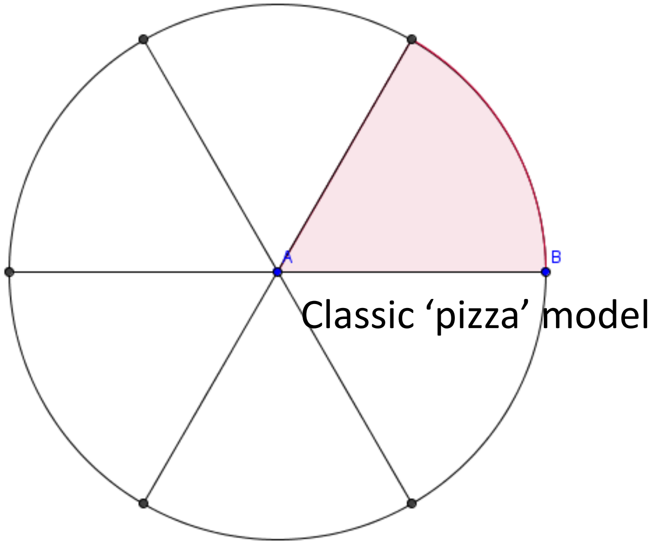
Jason: Sam, has Elliot split his board into quarters?

Sam: Um … yes … no …

From the outset, Jason faces a *fundamental* problem.

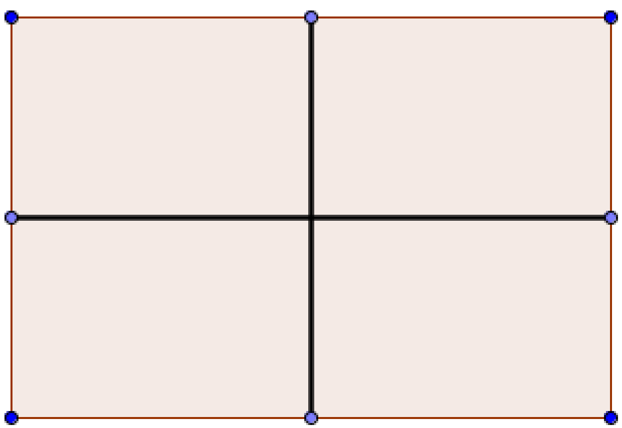
The pupils' previous experience of fractions has probably not prepared them even to begin to decide whether Elliot has 'split' his board into quarters. Why? Because in their experience of fractions until now, the *parts* have always been *congruent*.



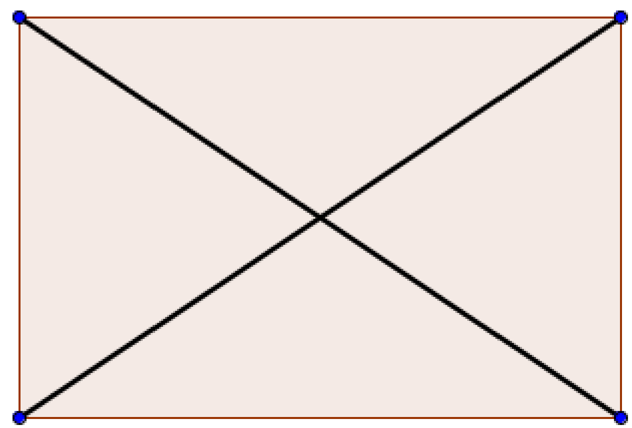


In Rebecca’s representation we can say that the four parts are ‘the same’ in the obvious sense they are the same shape and size (congruent)

In Elliot's representation, this is no longer the case, so *a different criterion for ‘sameness’* – equal area – becomes necessary.

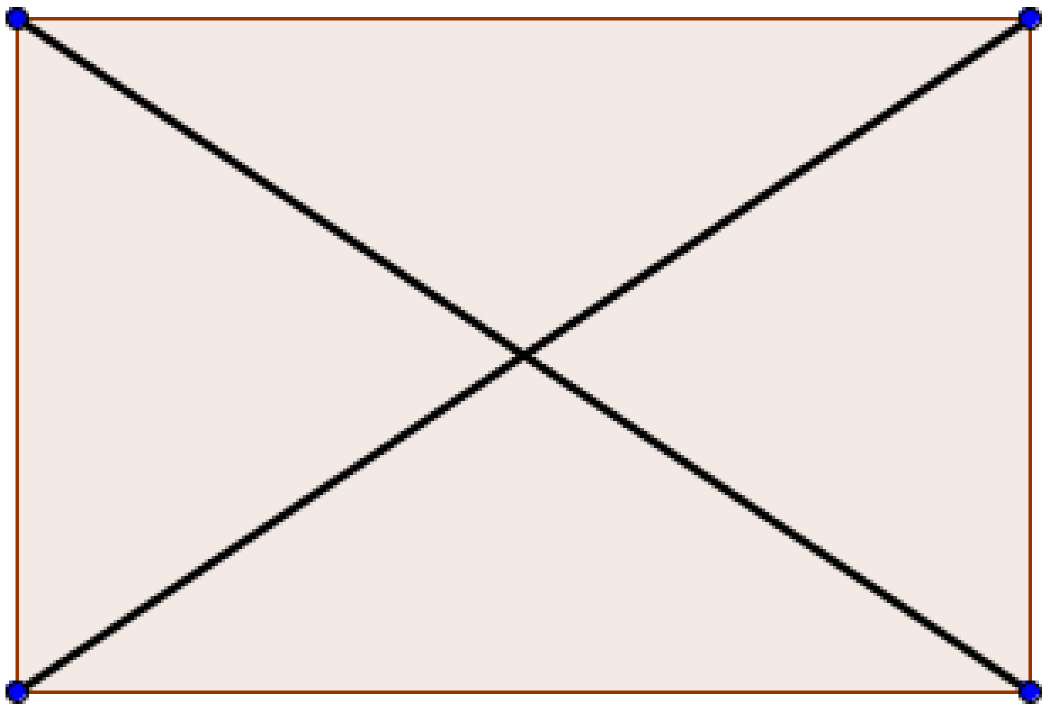


Rebecca’s representation



Elliot’s representation

Second: *is it actually true* that the areas of these non-congruent triangles are equal?

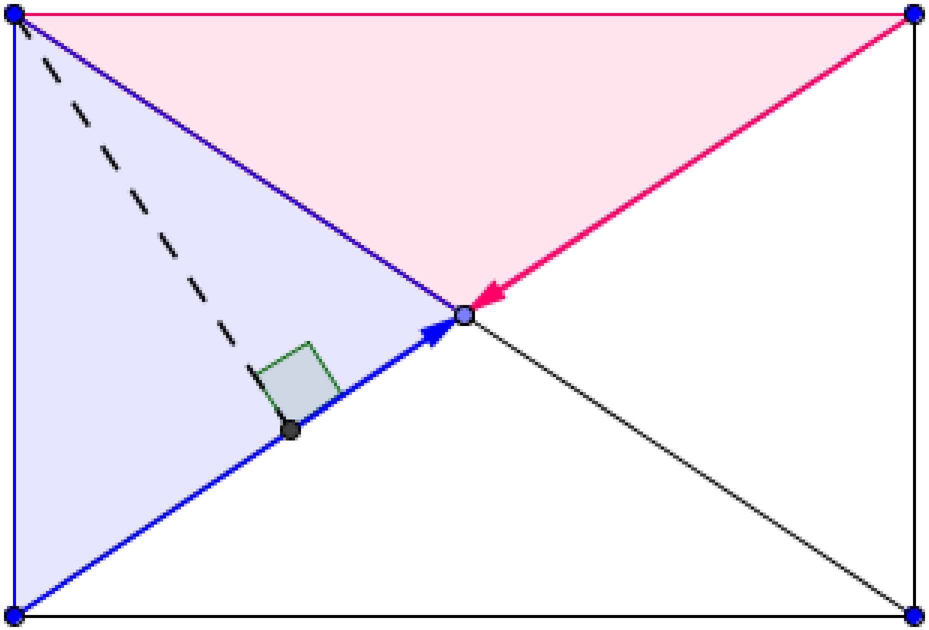


(do *you* know?)

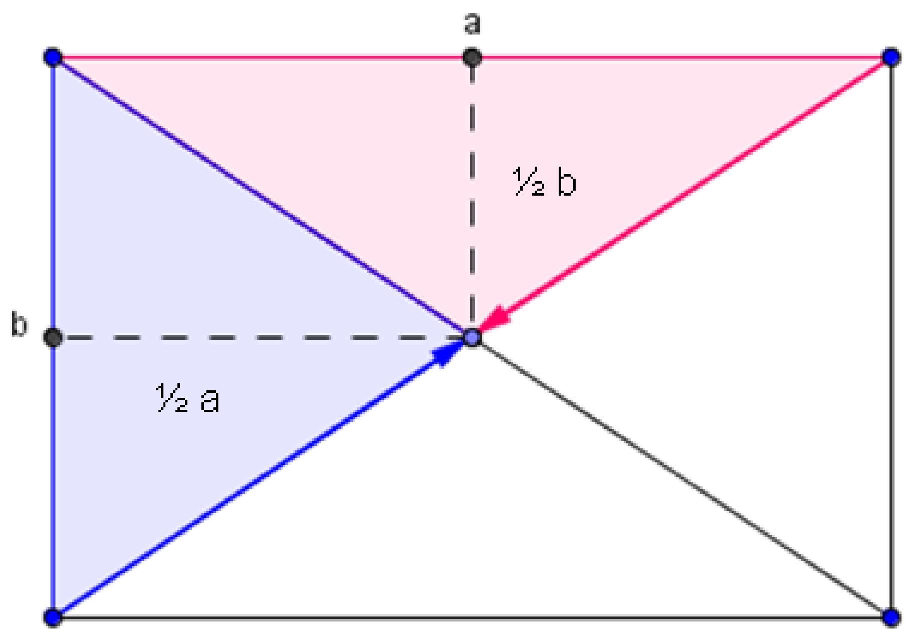
Are the areas of these non-congruent triangles equal?

In the first instance, Jason has to decide this for himself. He might use: Area of a triangle = ½ base x height

The two non-congruent triangles have the same (length base) and the *same* height.



Or – he might tackle it by algebra (I find that most ‘educated’ adults that I ask do this first)



Pink triangle: Area = ½ [a x ½ b]

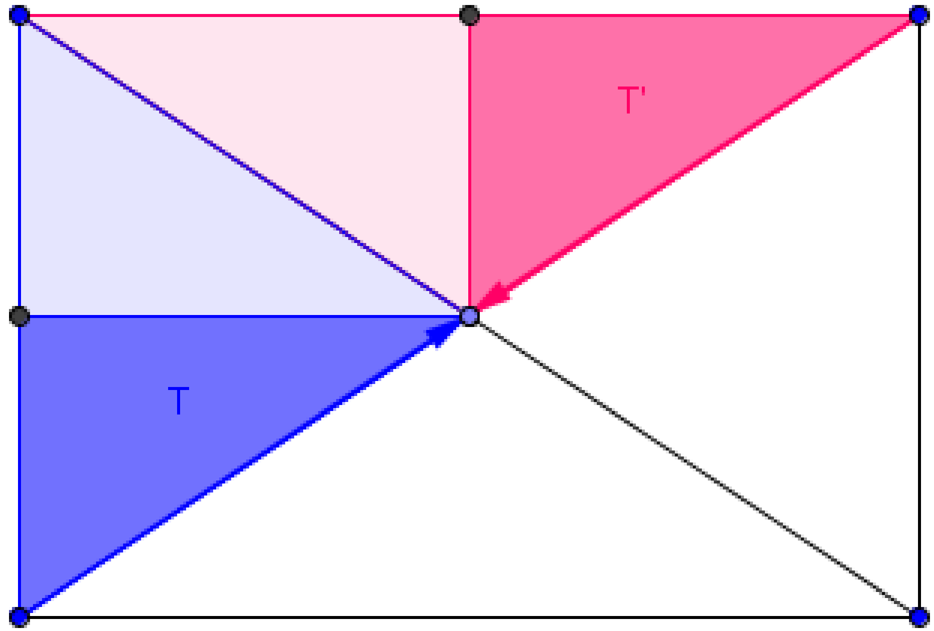
Blue triangle: Area = ½ [b x ½ a]

Third: Either way, Jason has solved his *mathematical* problem.

But there remains an as-yet unsolved mathematics-*pedagogical* problem, because:

Neither of the two ways of showing that the two non-congruent triangles have equal area is likely to be accessible by his class of 7-8 year-old pupils! He can *tell* them that the areas are equal, but how can he *justify* this claim?

A possible solution:



The equality of the areas is established by appeal to perception/transformation, without the need of algebra or any mensuration formulae.

To summarise: teaching elementary mathematics to young children brought to the surface demands on Jason's knowledge resources of three kinds.

Curricular – knowing what *representations* of equal fractions the children are likely to have experienced before, and that this one is different.

Mathematical – deciding whether Elliot's claim is true.

Pedagogical – what kinds of 'explanations' will be meaningful and convincing to these children.

Unsurprisingly, therefore, research shows that teachers with *more secure knowledge of mathematics and mathematics pedagogy* tend to be more effective teachers of mathematics.

Rowland, T., Martyn, S., Barber, P. and Heal, C. (2000) Primary teacher trainees' mathematics subject knowledge and classroom performance. *Research in Mathematics Education 2*, pp. 3-18

Hill, H. C., Rowan, B., & Ball, D. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371–406

Baumert, J. et al. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, 47 (1), 133-180.

But – this is <u>not</u> the same thing as saying that teachers with more advanced *qualifications* in mathematics (A level, BSc etc) necessarily teach primary mathematics better.

What matters is '*profound understanding of fundamental mathematics*' i.e. knowing the content and pedagogy of primary mathematics really well. This is not learned at school, but in *teachers' professional development*.

Ma, L. (1999) *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States,* Lawrence Erlbaum Associates, Mahwah, New Jersey

Conclusion

Teaching elementary mathematics is not straightforward!

Mathematics teacher knowledge is:

- multidimensional
- not ‘abstract’ but located in the professional (classroom) context
- embedded in knowledge of pedagogy and of children's’ thinking.

Knowledge for teaching primary mathematics: More than meets the eye

Tim Rowland

Advisory External Evaluator, *Enhancing Mathematical Learning through Talk*

3rd July 2015

University of East Anglia

HiST TRONDHEIM

UNIVERSITY OF CAMBRIDGE